A simple proof

Lin Zhong

Liu and Layland provide a sufficient condition for a set of periodic tasks to have a feasible fixed-priority schedule in [1]. Task i is denoted by C_i, the time to serve each of the periodic requests, and T_i, the period. Liu and Layland's theorem states that the set of m tasks are feasible if

$$\Sigma_{i=1}^{m} \frac{C_i}{T_i} \leq m(2^{1/m} - 1) \tag{1}$$

Bini, Buttazzo and Buttazzo provide a better (less pessimistic) sufficient condition in [2]. Their theorem states that the set of tasks are feasible if

$$\Pi_{i=1}^{m} (\frac{C_i}{T_i} + 1) \leq 2 \tag{2}$$

To prove the condition in Equation 2 is better (less pessimistic) than that in Equation 1, we only need to prove that if a set of tasks meet the latter, they also meet the former. That is, given Equation 1, we must prove Equation 2.

Proof.

$$\Sigma_{i=1}^{m} \frac{C_i}{T_i} \leq m(2^{1/m} - 1) \implies \Sigma_{i=1}^{m} (\frac{C_i}{T_i} + 1) \leq m \cdot 2^{1/m}$$

$$\implies \frac{\Sigma_{i=1}^{m} (\frac{C_i}{T_i} + 1)}{m} \leq 2^{1/m}$$

Given the *inequality of arithmetic and geometric means* [3], we have

$$[\Pi_{i=1}^{m} (\frac{C_i}{T_i} + 1)]^{1/m} \leq \frac{\Sigma_{i=1}^{m} (\frac{C_i}{T_i} + 1)}{m} \leq 2^{1/m}$$

$$\implies \Pi_{i=1}^{m} (\frac{C_i}{T_i} + 1) \leq 2$$

□

References

[1] Chung Laung Liu and James W Layland. Scheduling algorithms for multiprogramming in a hard-real-time environment. *Journal of the ACM (JACM)*, 20(1):46–61, 1973.

[2] Enrico Bini, Giorgio C Buttazzo, and Giuseppe M Buttazzo. Rate monotonic analysis: the hyperbolic bound. *IEEE Transactions on Computers*, 52(7):933–942, 2003.

[3] Wikipedia. Inequality of arithmetic and geometric means. `https://en.wikipedia.org/wiki/Inequality_of_arithmetic_and_geometric_means`.